EXPONENTIAL HADRONIC SPECTRUM AND QUARK LIBERATION

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The exponentially increasing spectrum proposed by Hagedorn is not necessarily connected with a limiting temperature, but it is present in any system which undergoes a second order phase transition. We suggest that the "observed" exponential spectrum is connected to the existence of a different phase of the vacuum in which quarks are not confined.

It has been shown by Hagedorn [1,2] that the statistical bootstrap hypothesis leads to an exponentially increasing spectrum of hadronic states. As a consequence of this there is a critical temperature T_c which was interpreted as a limiting temperature, i.e. hadronic matter cannot exist for $T > T_c$.

In the present note we show that a bootstrap hypothesis similar to that formulated by Hagedorn is actually satisfied in any model where hadronic matter has a second order phase transition[‡1]. This means that models which have Hagedorn-type exponential spectrum may either lead to a second order phase transition for hadronic matter, or to a limiting temperature. We will argue that the first alternative is realized in quark containement model [3–5], and that these models will develop a phase transition to a state where quarks are free to move in space (quark liberation). As a corollary of our considerations, it follows that "quark containement" models should lead to a Hagedorn-type hadron spectrum. This result is in fact already known to hold in the bag model [4].

The main difference of our treatment with that which is standard in previous papers on the statistical bootstrap is that we will work in the thermodynamic ($V \to \infty$) limit [1,2]. The main reason for doing so is that we wish to interpret hadron levels as levels effectively observed in scattering processes. This means that the level density has to be defined in terms of the S-matrix. This has in fact been done by Dashen, Ma and Bernstein [6] we obtain

$$w(E) = \mathrm{Tr}\left[S^+(E)\frac{\overleftrightarrow{\partial}}{\partial E}S(E)\right]_c \cdot (4\pi i)^{-1}. \tag{1}$$

In the narrow width limit $w(E)$ is simply connected to the density of resonant levels. The free energy density in the infinite volume limit, $F(\beta)$ can be written in terms of $w(E)$ as:

$$F(\beta) = \int dE\, w(E) \exp(-\beta E), \tag{2}$$

where $\beta = (kT)^{-1}$ [‡2].

Another motivation for the use of a thermodynamical limit is that any finite volume treatment would miss a possible phase transition [7] and a fortiori a second order transition which is characterized by the appearance or disappeance of long range order.

We can start by defining the partition for finite volume

$$Z(\beta, V) = \int dE\, \sigma(E, V) \exp(-\beta E), \tag{3}$$

where $\sigma(E, V)$ is the density of states in volume V. In the thermodynamical limit we can define the free energy density as

$$F(\beta) = \lim_{V\to\infty} \frac{1}{V} \ln Z(\beta, V), \tag{4}$$

[‡1] According to current usage, we call "second order transition" any transition where thermodynamical quantities are actually singular at the transition temperature.

[‡2] Eq. (2) can be valid for large β, i.e. when T is smaller than any transition temperature.

and a partition function normalized to a reference volume V_0:

$$Z(\beta) \equiv \exp\{V_0 F(\beta)\} = \lim_{V\to\infty} [Z(\beta, V)]^{V_0/V}. \qquad (5)$$

For large β it is reasonable to assume that $Z(\beta)$ can be expressed as

$$Z(\beta) = \int \mathrm{d}E\, \rho(E) \exp(-\beta E). \qquad (6)$$

We note that, far from a transition, if we choose $V^0 \gg \xi^3$, where ξ is the correlation lenght, $Z(\beta)$ defined in (5) practically coincides with $Z(\beta, V_0)$ apart from surface terms which are $O(\xi\, V_0^{-1/3})$ [7]. Near the transition, where $\xi \to \infty$, $Z(\beta)$ gives a physically acceptable definition of "partition function relative to a volume V_0".

A formal relation between $w(E)$ and $\rho(E)$ is obtained by comparing eqs. (5), (6) and (2):

$$\rho(E) = \sum_{N=0}^{\infty} \frac{V_0^{-N}}{N!} \int \delta \left(\sum_{i=1}^{N} E_i - E\right) \prod_{i=1}^{N} (w(E_i)\, \mathrm{d}E_i). \qquad (7)$$

Following Hagedorn [1] we will impose the bootstrap condition:

$$\lim_{E\to\infty} \frac{\ln w(E)}{\ln \rho(E)} = 1. \qquad (8)$$

This relation can be satisfied if, for large E, $w(E) \to E^{\alpha-3} \exp(E\,\beta_c)$ with $\alpha < 2$ [1,8]. Under these hypothesis $F(\beta)$ will behave, for $\beta \approx \beta_c$, like[‡3]

$$F(\beta) = A(\beta - \beta_c)^{2-\alpha} + \text{less singular terms}. \qquad (9)$$

Substituting (9) in eq. (5) we find

$$Z(\beta) = \exp(V_0 A(\beta-\beta_c)^{2-\alpha} + \ldots)$$

$$= \exp\{V_0 F(\beta_c)\}\ [1 + V_0 A(\beta-\beta_c)^{2-\alpha} \qquad (10)$$

$$+ \tfrac{1}{2} V_0^2 A^2 (\beta-\beta_c)^{2(2-\alpha)} + \text{less singular terms}].$$

Taking the Laplace transform of (10) we find

$$\rho(E) \propto E^{\alpha-3}(1 + O(E^{\alpha-2})) \exp(\beta_c E), \qquad (11)$$

so that eq. (8) is satisfied.

The singularity of the free energy displayed by eq. (9) is typical of a second order transition.

Three dimensional systems like the Ising model, liquid helium and ferromagnetic materials have $\alpha \approx 0$, so that the bootstrap condition is satisfied.

Only if $\alpha \geqslant 1$ the internal energy density

$$U(\beta) = \frac{\mathrm{d}}{\mathrm{d}\beta} F(\beta)$$

becomes divergent at $\beta = \beta_c$ and $T_c = 1/k\beta_c$ is a limiting temperature.

For $\alpha < 1$ $U(\beta)$ reaches a finite limit at $\beta = \beta_c$. At greater temperature all thermodynamical quantities remain finite, but the integral representations, eqs. (2) and (5) are not any more valid.

There is some indication [1] of an exponential spectrum obtained by level counting. We cannot however expect this method to have sufficient accuracy to obtain a reliable value of α [8] amd therefore decide between a limiting temperature and a phase transition.

Dual models lead to a spectrum of the exponential type [9]. The value of α depends critically from the details of the model.

Results by Huang and Weinberg [10] indicate that $\alpha = 2$ in the simple Veneziano model, and that smaller values of α would be obtained in more complex models[‡4].

The currently accepted interpretation of the properties of hadronic matter is based on the "realistic" quark model where quarks are permanently confined in hadrons. We expect models of this kind to give rise to a phase transition at a temperature $kT \approx m_\pi$, the high temperature phase being one where quarks can move freely in space.

It is interesting to see how this transition arises in two different quark containement models.

The first is a model proposed by one of us [5] where the containement is provided by a gauge field B^μ, the gauge symmetry being spontaneously broken. If quarks are monopoles of B^μ, isolated quarks have infinite energy as a consequence of the conservation law for magnetic flux and the Meissner effect [11]. Quark antiquark pairs can exist with a finite energy.

In this model we expect [12] that increasing the temperature the gauge symmetry will be restored, the Meissner effect will disappear, and quarks will be liberated.

The second is the MIT bag model [4]. In this model

[‡3] The case $\alpha = 2$ was treated in ref. [1] with similar results, While the argument obviously fails for $\alpha > 2$.

[‡4] These results do not take into account the presence of ghosts.

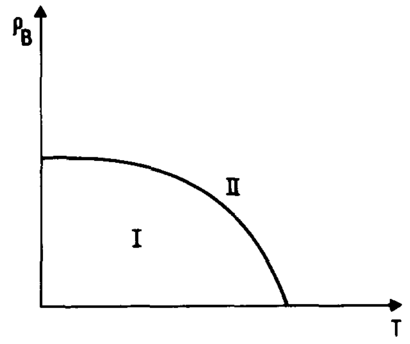

Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

a hadron consists of a bag inside which quarks are confined. If many hadrons are present, space is divided into two regions: the "exterior" and the "interior". At low temperature the hadron density is low, and the "interior" is made up of disconnected islands (the hadrons) in a connected sea of "exterior". By increasing the temperature, the hadron density increases, and so does the portion of space belonging to the "interior". At high enough temperature we expect a transition to a new situation, where the "interior" has fused into a connected region, with isolated ponds and lakes of exterior. Again, in the high temperature state, quarks can move throughout space. We note that this picture of the quark liberation is very close to that of the droplet model of second order phase transitions [13].

We expect the same transition to be also present at low temperature but high pressure, for the same reason, i.e. we expect a phase diagram of the kind indicated in fig. 1. The true phase diagram may actually be substantially more complex, due to other kinds of transitions, such as, e.g. those considered by Omnes [14].

We note finally that, although the two alternatives (phase transition or limiting temperature) give rise to similar forms for the hadronic spectrum, the equation of state for high densities is radically different. In the first case we may expect the equation of state to become asymptotically similar to that of a free Fermi gas, while the limiting temperature case leads to an extremely "soft" equation of state [15]. This difference has important astrophysical implications [16].

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